

Free response review No calculator

1. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x-2} & \text{for } 3 \leq x \leq 6 \\ 8-x & \text{for } 6 < x \leq 8 \end{cases}$$

- a) Is f continuous at $x = 6$? Explain why or why not.
- b) Find the average value of $f(x)$ on the closed interval $3 \leq x \leq 8$.

- c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x-2} & \text{for } 3 \leq x \leq 6 \\ mx + 4 & \text{for } 6 < x \leq 8 \end{cases}$$

where k and m are constants. If g is differentiable at $x = 6$, what are the values of k and m ?

2. Consider the curve given by $x^2 + 3y^2 = 12 + 3xy$.

a) Show that $\frac{dy}{dx} = \frac{3y-2x}{6y-3x}$

b) Show that there is a point P with x-coordinate 6 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at point P? Justify your answer.

d) Find all points on the curve whose x-coordinate is 0 and write an equation for the tangent line at each of these points.

e) Find the x-coordinates of each point on the curve where the tangent line is vertical.

