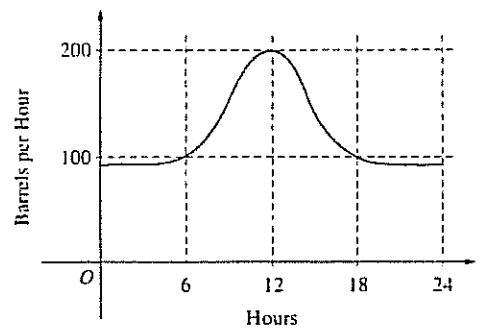


AB Review 06, No Calculator Permitted, except ~~on~~ *where indicated!*
 Do all work on separate notebook paper

1. Let f and g be differentiable functions with the following properties:
 (i) $g(x) > 0$ for all x
 (ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $f'(x) = f(x)g'(x)$, then $f(x) =$
 (A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown at right. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?
 (A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800



3. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

4. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

5. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

6. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$ (B) $\cos(e^{-x}) + e^{-x}$ (C) $\cos(e^{-x}) - e^{-x}$ (D) $e^{-x} \cos(e^{-x})$ (E) $-e^{-x} \cos(e^{-x})$

7. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

8. What are all the values of k for which $\int_{-3}^k x^2 dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

9. The average value of the function $f(x) = 2e^{(x-3)}$ on the interval $[1, 6]$ is

- (A) $\frac{e^3}{3}$ (B) $2e^3 - 2e^{-2}$ (C) $\frac{e^3 - e^2}{3}$ (D) $e^3 + e^{-5}$ (E) $\frac{2e^3}{5} - \frac{2e^{-2}}{5}$

10. A rectangle has its base on the x -axis and both its other vertices on the positive portion of the parabola $y = 3 - 4x^2$. What is the maximum possible area of this rectangle?

- (A) $\frac{3\sqrt{6}}{4}$ (B) $\frac{3\sqrt{15}}{5}$ (C) $\frac{3\sqrt{15}}{10}$ (D) 2 (E) $\frac{3}{2}$

11. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

- (A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$ (D) $\tan x + 5$ (E) $\tan x + 5e^x$

12. (Calculator Permitted) The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1, 1]$ is

- (A) 0.70 (B) 0.75 (C) 0.80 (D) 0.85 (E) 0.90

13. If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1, 4]$ is

- (A) $-\frac{1}{4}$ (B) $\frac{1}{2} \ln 2$ (C) $\frac{2}{3} \ln 2$ (D) $\frac{2}{5}$ (E) 2

14. $\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$

- (A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

$$f(x) = \begin{cases} cx + d & \text{for } x \leq 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

15. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

- (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

16. Determine the y -intercept of the tangent line to the curve $y = \sqrt{x^2 + 24}$ at $x = 5$.

- (A) $\frac{24}{7}$ (B) $-\frac{72}{49}$ (C) $\frac{48}{49}$ (D) $\frac{44}{7}$ (E) $\frac{88}{49}$

17. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \leq x \leq 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

- (A) None (B) I only (C) III only (D) I and III only (E) II and III only

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

18. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x) dx$?
- (A) 8 (B) 12 (C) 16 (D) 24 (E) 32

19. When the region enclosed by graphs of $y = x$ and $y = 4x - x^2$ is revolved about the y -axis, the volume of the solid generated is given by

(A) $\pi \int_0^3 (x^3 - 3x^2) dx$ (B) $\pi \int_0^3 (x^2 - (4x - x^2)^2) dx$ (C) $\pi \int_0^3 (3x - x^2)^2 dx$
(D) $2\pi \int_0^3 (x^3 - 3x^2) dx$ (E) $2\pi \int_0^3 (3x^2 - x^3) dx$

20. (Calculator Permitted) Let $F(x) = \cos(2x) + e^{-x}$. For what value of x on the interval $[0, 3]$ will F have the same instantaneous rate of change as the average rate of change of F over the interval?
- (A) 1.542 (B) 1.610 (C) 1.678 (D) 1.746 (E) 1.814

21. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f , that approximation is
- (A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

22. (Calculator Permitted) If $f'(x) = \frac{x^2}{1+x^5}$ and $f(1) = 3$, then $f(4) =$
- (A) 2.988 (B) 3 (C) 3.016 (D) 3.376 (E) 3.629

23. The base of a solid is the region in the first quadrant enclosed by the graphs of $y = 2 - x$ and the coordinate axes. If every cross section of the solid perpendicular to the y -axis is a square, the volume of the solid is given by

(A) $\pi \int_0^2 (2-y)^2 dy$ (B) $\int_0^2 (2-y)^2 dy$ (C) $\pi \int_0^{\sqrt{2}} (2-x^2)^2 dx$ (D) $\int_0^{\sqrt{2}} (2-x^2)^2 dx$ (E) $\int_0^{\sqrt{2}} (2-x^2) dx$

24. $\lim_{h \rightarrow 0} \frac{3\left(\frac{1}{2} + h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h} =$

- (A) 0 (B) 1 (C) $\frac{15}{16}$ (D) the limit does not exist (E) the limit cannot be determined