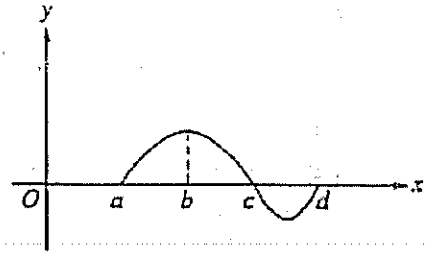
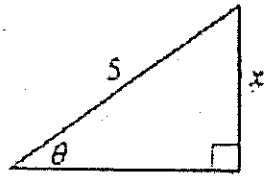
AB Review 03, No calculator.
Do all work on separate notebook paper (except for #5).

1. The graph of f is shown in the figure on the right. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

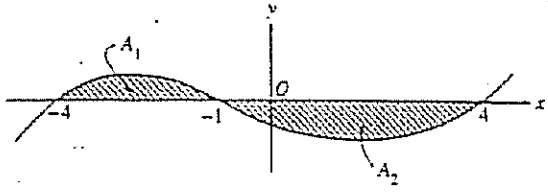


(A) a (B) b (C) c (D) d (E) It cannot be determined from the information given

2. In the triangle shown on the right, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing, in units per minute, when $x=3$ units?



(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12



3. The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx-2\int_{-1}^{4} f(x)\,dx=$

(A) A_1 (B) A_1-A_2 (C) $2A_1-A_2$ (D) A_1+A_2 (E) A_1+2A_2

x	0	1	2	3
$f''(x)$	5	0	−7	4

4. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0,2)$.

(B) f is decreasing on the interval $(0,2)$.

(C) f has a local maximum at $x=1$.

(D) The graph of f changes concavity in the interval $(0,2)$.

5. $\int_1^4 \frac{dx}{\sqrt{16-x^2}}=$

(A) $\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (B) $-\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (C) $\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

(D) $-4\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (E) $4\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$.

6. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4\left(x^2-4\right)^2}+C$ (B) $\frac{1}{2\left(x^2-4\right)}+C$ (C) $\frac{1}{2}\ln\left|x^2-4\right|+C$ (D) $2\ln\left|x^2-4\right|+C$ (E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

7. The position of a particle moving along the x-axis at time t is given by $x(t)=\sin^2(4\pi t)$. At which of the following values of t will the particle change direction?

I. $t=\frac{1}{8}$

II. $t=\frac{1}{6}$

III. $t=1$

IV. $t=2$

(A) II, III, and IV (B) I and II (C) I, II, and III (D) III and IV (E) I, III, and IV

8. The region enclosed by the x-axis, the line $x=3$, and the curve $y=\sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $3\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

9. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

10. If $\frac{dy}{dx}=2y^2$ and if $y=-1$ when $x=1$, then when $x=2$, $y=$

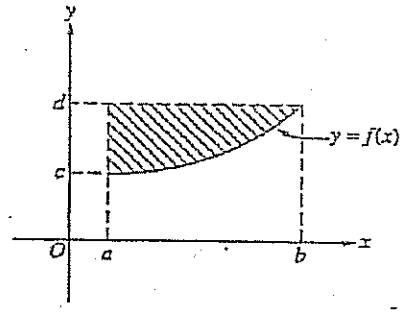
(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

11. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change, in feet per minute, of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ (B) $-\frac{7}{24}$ (C) $\frac{7}{24}$ (D) $\frac{7}{8}$ (E) $\frac{21}{25}$

12. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x



13. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b \big(d-f(x)\big)\,dx$ (C) $f'(b)-f'(a)$

(D) $(b-a)\big[f(b)-f(a)\big]$ (E) $(d-c)\big[f(b)-f(a)\big]$

14. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $-\frac{x^2}{1+2y^2}$

15. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx=$

(A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$ (D) $\ln\sqrt{x^3+1}+C$ (E) $\ln\left(x^3+1\right)+C$

16. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

17. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

18. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

19. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

(A) $9t^2+1$ (B) $3t^2-2t+4$ (C) t^3-t^2+4t+6 (D) $t^3-t^2+9t-20$ (E) $36t^3-4t^2-77t+55$

20. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

21. What is the minimum value of $f(x)=x\ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

Review of Concepts – Increasing-decreasing intervals; concavity intervals; inflection points

Given: f' equation (no calculator)

1. How do you find the intervals for which the function is increasing?

2. How do you find the relative maxima and relative minima?

3. How do you find the intervals for which the function is concave down?

4. How do you find the points of inflection?

5. How do you find the intervals where the function is both concave up and negative?

Given: f(x): $15x^4 - 60x^2$. Answer 1-5 by using this function. Remember: This is not the derivative function. Use the strategies you outlined above to this function. Leave your answers in radical form if necessary. No calculator.