16. The functions f and g are continuous for all real numbers. The table below gives values of the functions at selected values of x. The function h is given by $h(x)=g(f(x))+2$.

IVT

x	$f(x)$	$g(x)$
1	3	4
3	9	−10
5	7	5
7	11	25

Explain why there must be a value w for $1<w<5$ such that $h(w)=0$

17. The functions f and g are continuous for all real numbers. The function h is given by $h(x)=f(g(x))-x$. The table below gives values of the functions at selected values of x. Explain why there must be a value of u for $1<u<4$ such that $h(u)=-1$.

IVT

x	1	2	3	4
$f(x)$	0	8	−3	6
$g(x)$	3	4	1	2

3. Determine if the function $f(x) = x^3 - x - 1$ satisfies the hypothesis of the MVT on $[-1, 2]$. If it does, find all possible values of c satisfying the conclusion of the MVT.

(A) $-\frac{1}{2}$ (B) $-1, 1$ (C) $\frac{1}{2}$ (D) 0 (E) 1 (F) -1 (G) hypothesis not satisfied

4. Determine if the function $f(x) = x + x^{2/3}(1-x)^{1/3}$ satisfies the hypothesis of the MVT on $[0,1]$. If it does, find all possible values of c satisfying the conclusion of the MVT.

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{2}{3}$ (F) hypothesis not satisfied

5. Which of the following functions below satisfy the hypothesis of the MVT?

I. $f(x) = \frac{1}{x+1}$ on $[0,2]$

II. $f(x) = x^{1/3}$ on $[0,1]$

III. $f(x) = |x|$ on $[-1,1]$

(A) I only (B) I and II only (C) I and III only (D) III only (E) none of them (F) all of them

13. Determine the values of a, b, and c such that the function f satisfies the hypothesis of the MVT on the interval $[0,3]$.

$$f(x)=\begin{cases}1, & x=0\\ ax+b, & 0<x\le 1\\ x^2+4x+c, & 1<x\le 3\end{cases}$$

MVT

14. Suppose that we know that $f(x)$ is continuous and differentiable on $[6,15]$. Let's also suppose that we know that $f(6)=-2$ and that $f'(x)\le 10$ for all $x\in[6,15]$. What is the largest possible value for $f(15)$?

MVT