AB Fall Final Exam Review 2013

Name_____ Date_____Period_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem. NO CALCULATOR.

1) Find an equation of the tangent line to the graph of $y = 2\sqrt{x} - x + 7$ at the point $(4, 7)$. 1) _____

A) $y = 7$ B) $y = -\frac{1}{2}x + 9$ C) $y = \frac{1}{2}x - 9$ D) $y = -\frac{1}{2}x + 7$

2) Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec. Find the watermelon's speed at the instant $t = 6$ sec. 2) _____

A) 194 ft/sec B) 96 ft/sec C) 192 ft/sec D) 97 ft/sec

3) At time t, the position of a body moving along the s-axis is $s = t^3 - 27t^2 + 240t$ m. Find the body's acceleration each time the velocity is zero. 3) _____

A) $a(10) = -6$ m/sec^2, $a(8) = 6$ m/sec^2 B) $a(10) = 6$ m/sec^2, $a(8) = -6$ m/sec^2

C) $a(10) = 0$ m/sec^2, $a(8) = 0$ m/sec^2 D) $a(20) = 120$ m/sec^2, $a(16) = 20$ m/sec^2

4) At time $t \geq 0$, the velocity of a body moving along the s-axis is $v = t^2 - 10t + 9$. When is the body moving backward? 4) _____

A) $1 < t < 9$ B) $0 \leq t < 1$ C) $0 \leq t < 9$ D) $t > 9$

5) The position of a particle moving along a coordinate line is $s = \sqrt{3 + 6t}$, with s in meters and t in seconds. Find the particle's velocity at $t = 1$ sec. 5) _____

A) 2 m/sec B) 1 m/sec C) $-\frac{1}{3}$ m/sec D) $\frac{1}{6}$ m/sec

Find all points where the function is discontinuous.

6) 6) _____

$$f(x) = \begin{cases} \frac{x(x^2 - 4)}{x + 2}, & x \neq -2 \\ -4, & x = -2 \end{cases}$$

A) $x = -4$ B) Continuous for all x

C) $x = 2$ D) $x = -2$

7) $f(x) = \begin{cases} 0, & x < 0 \\ x^2 - 4x, & 0 \leq x \leq 4 \\ 4, & x > 4 \end{cases}$ 7) _____

A) $x = 4$ B) $x = 0$ and $x = 4$ C) $x = 0$ D) Nowhere

Find dy/dx.

8) $y = 7xe^x - 7e^x$ 8) _____

A) $7e^x$ B) $7xe^x$ C) $7xe^x + 14e^x$ D) $7x$

9) $y = \frac{7x + 3}{6x - 2}$ 9) _____

A) $\frac{84x + 4}{(6x - 2)^2}$ B) $-\frac{32x}{(6x - 2)^2}$ C) $\frac{4}{6x - 2}$ D) $-\frac{32}{(6x - 2)^2}$

10) $y = \ln(\ln 5x)$ 10) _____

A) $\frac{1}{x}$ B) $\frac{1}{5x}$ C) $\frac{1}{\ln 5x}$ D) $\frac{1}{x \ln 5x}$

11) $y = \cot(6x - 3)$ 11) _____

A) $6 \cot(6x - 3) \csc(6x - 3)$ B) $-6 \csc^2(6x - 3)$

C) $-\csc^2(6x - 3)$ D) $-6 \sec^2(6x - 3)$

12) $y = 6^{\cos x}$ 12) _____

A) $6^{\cos x} \ln 6$ B) $6^{\cos x}$

C) $-6^{\cos x} \ln 6 \sin x$ D) $6^{\cos x} \ln 6 \sin x$

13) $y = \log(2x - 9)$ 13) _____

$\log u = \frac{\ln u}{\ln 10}$

$(\log u)' = \frac{u'}{u(\ln 10)}$

A) $\frac{2x - 9}{2 \ln 10}$ B) $\frac{2}{\ln 10}$ C) $\frac{1}{(2x - 9) \ln 10}$ D) $\frac{2}{(2x - 9) \ln 10}$

14) $y = (2x - 2)(2x^3 - x^2 + 1)$ 14) _____

A) $12x^3 + 18x^2 - 6x + 2$ B) $16x^3 - 6x^2 + 18x + 2$

C) $16x^3 - 18x^2 + 4x + 2$ D) $4x^3 + 6x^2 - 18x + 2$

15) $s = t^3 \tan t$ 15) _____

A) $t^3 \sec^2 t + 3t^2 \tan t$ B) $3t^2 \sec^2 t$

C) $t^3 \sec t \tan t + 3t^2 \tan t$ D) $-t^3 \sec^2 t + 3t^2 \tan t$

Find the extreme values of the function on the interval and where they occur.

16) $g(x) = -x^2 + 11x - 30$ on $5 \le x \le 6$ 16) _____

A) Maximum value is $\frac{1}{4}$ at $x = \frac{13}{2}$; minimum value is 0 at $x = 6$ and 0 at $x = 5$

B) Maximum value is $\frac{241}{4}$ at $x = \frac{11}{2}$; minimum value is 0 at $x = 6$ and 0 at $x = 5$

C) Maximum value is $\frac{1}{4}$ at $x = \frac{11}{2}$; minimum value is 0 at $x = 6$ and 0 at $x = 5$

D) Maximum value is $\frac{5}{4}$ at $x = \frac{13}{2}$; minimum value is 0 at $x = 6$ and 0 at $x = 5$

Determine the limit algebraically, if it exists.

17) $\lim_{x \to 7} \frac{x^2 + 2x - 63}{x - 7}$ 17) _____

A) 2 B) 0 C) Does not exist D) 16

Find the intervals on which the function is continuous.

27) $y = \ln(3x - 1)$ 27) _____

A) $\left[\frac{1}{3}, \infty\right)$ B) $\left(-\infty, \frac{1}{3}\right]$ C) $\left[-\frac{1}{3}, \infty\right)$ D) $\left[\frac{1}{3}, \infty\right)$

Provide an appropriate response.

28) If $x^3 \le f(x) \le x$ for x in $[-1,1]$, find $\lim_{x \to 0} f(x)$ if it exists. 28) _____

A) -1 B) 1 C) Does not exist D) 0

Find the derivative at each critical point and determine the local extreme values.

29) $y = x^{2/3}(x^2 - 16); x \ge 0$ 29) _____

A)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	Undefined	local max	0
$x = 2$	0	minimum	−19.048813

B)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	0	maximum	0
$x = 2$	0	minimum	−19.048813

C)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	Undefined	local max	0
$x = 2$	0	minimum	31.748021

D)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	Undefined	local max	4
$x = 2$	0	minimum	−19.048813

Find dy/dx by implicit differentiation. If applicable, express the result in terms of x and y.

30) $\cos xy + x^5 = y^5$ 30) _____

A) $\frac{5x^4 - x \sin xy}{5y^4}$ B) $\frac{5x^4 + x \sin xy}{5y^4}$ C) $\frac{5x^4 + y \sin xy}{5y^4 - x \sin xy}$ D) $\frac{5x^4 - y \sin xy}{5y^4 + x \sin xy}$

31) $2y^2 + 7x^2 - 13 = 0$ 31) _____

A) $\frac{-7x}{2y}$ B) $\frac{-7x^2}{4y}$ C) $\frac{-14x + 13}{4y}$ D) $\frac{-7x}{2}$

Example: $\sin(xy) = 3$ find $\frac{dy}{dx}$ · derivative of 3

$\downarrow$ $\downarrow$

$(\sin(xy))' = \cos(xy)\left[y + xy'\right] = 0$

$y\cos(xy) + xy'\cos(xy) = 0$

$xy'\cos(xy) = -y\cos(xy)$

$y' = \frac{-y\cos(xy)}{x\cos(xy)}$

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \to d-} f(x)$, (b) $\lim_{x \to d+} f(x)$, and (c) $\lim_{x \to d} f(x)$ for the given function f and number d.

15) 15) _____

$$f(x) = \begin{cases} 4x - 6, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -7x + 10, & \text{for } x > 1 \end{cases}$$

$d = 1$

A) (a) −2
(b) 3
(c) Does not exist

B) (a) 3
(b) −2
(c) Does not exist

C) (a) 3
(b) −2
(c) 1

D) (a) −2
(b) 3
(c) 1

Solve the problem. 16-20 (calc. ok) [Related rates / Optimization review]

16) Suppose $c(x) = x^3 - 18x^2 + 20{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items. 16) _____

A) 11 items B) 9 items C) 8 items D) 10 items

17) The radius of a right circular cylinder is increasing at the rate of 5 in./s, while the height is decreasing at the rate of 8 in./s. At what rate is the volume of the cylinder changing when the radius is 15 in. and the height is 20 in.? 17) _____

A) -80 in.^{3}/s B) 1200π in.^{3}/s C) -300 in.^{3}/s D) -300π in.^{3}/s

18) A man flies a kite at a height of 50 m. The wind carries the kite horizontally away from him at a rate of 10 m/sec. How fast is the distance between the man and the kite changing when the kite is 130 m away from him? 18) _____

A) 10 m/sec B) 10.9 m/sec C) 9.2 m/sec D) 51 m/sec

19) A piece of molding 153 cm long is to be cut to form a rectangular picture frame. What dimensions will enclose the largest area? Round to the nearest hundredth, if necessary. 19) _____

A) 12.37 cm × 12.37 cm
B) 30.6 cm × 30.6 cm
C) 38.25 cm × 38.25 cm
D) 12.37 cm × 38.25 cm

20) Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec. Find the watermelon's speed at the instant $t = 4$ sec. 20) _____

A) 128 ft/sec B) 65 ft/sec C) 130 ft/sec D) 64 ft/sec

21) A man 6 ft tall walks at a rate of 7 ft/s away from a lamppost that is 18 ft high. At what rate is the length of his shadow changing when he is 70 ft away from the lamppost? 21) _____

A) $\frac{7}{4}$ ft/s B) $\frac{7}{2}$ ft/s C) $\frac{7}{8}$ ft/s D) $\frac{245}{3}$ ft/s

22) At time t, the position of a body moving along the s-axis is $s = t^3 - 9t^2 + 24t$ m. Find the body's acceleration each time the velocity is zero. 22) _____

A) $a(2) = 6$ m/sec^2, $a(4) = -6$ m/sec^2
B) $a(2) = -6$ m/sec^2, $a(4) = 6$ m/sec^2
C) $a(2) = 0$ m/sec^2, $a(4) = 0$ m/sec^2
D) $a(4) = 24$ m/sec^2, $a(8) = 4$ m/sec^2

Determine the values of x for which the function is differentiable.

39) $y = \dfrac{1}{x^2 - 121}$ 39) _____

A) All reals except –11 and 11
B) All reals except 11
C) All reals except 121
D) All reals

Find the indicated limit.

40) $\lim\limits_{x \to 0^-} \dfrac{11x}{|x|}$ 40) _____

A) 11 B) –11 C) 0 D) Does not exist

Find the limit, if it exists.

41) $\lim\limits_{x \to -\infty} \dfrac{-12x^2 + 8x + 9}{-15x^2 + 2x + 8}$ 41) _____

A) 1 B) $\frac{9}{8}$ C) ∞ D) $\frac{4}{5}$

42) $\lim\limits_{x \to -\infty} \dfrac{4x^3 + 3x^2}{x - 6x^2}$ 42) _____

A) 4 B) $-\frac{1}{2}$ C) ∞ D) $-\infty$

Find the slope of the line tangent to the curve at the given value of x.

43) $f(x) = \dfrac{-2}{x + 5}; x = -2$ 43) _____

A) $-\frac{2}{9}$ B) $\frac{2}{9}$ C) $\frac{4}{9}$ D) $-\frac{4}{9}$

44) $f(x) = -2x^2 + 6x; x = 6$ 44) _____

A) 12 B) –3 C) –18 D) –108

Suppose that the functions f and g and their derivatives with respect to x have the following values at the given values of x. Find the derivative with respect to x of the given combination at the given value of x.

45) 45) _____

x	f(x)	g(x)	f '(x)	g '(x)
3	1	16	8	3
4	–3	3	5	–4

$\sqrt{f(x) + g(x)}$ at $x = 3$

A) $-\frac{1}{2\sqrt{17}}$ B) $\frac{11}{\sqrt{17}}$ C) $\frac{11}{2\sqrt{17}}$ D) $\frac{1}{2\sqrt{17}}$

Find a value for a so that the function f(x) is continuous.

46) $f(x) = \begin{cases} x^2 - 5, & x < 4 \\ 5ax, & x \geq 4 \end{cases}$ 46) _____

A) $a = 11$ B) $a = \frac{4}{5}$ C) $a = 9$ D) $a = \frac{11}{20}$

47. use the table provided on #45.

$(f(g(4))' = ?$

u-sub integration review

4. Evaluate the definite integral $\int_0^1 (4-2x)e^{8x-2x^2}\,dx$

(A) $\frac{1}{2}\left(e^6-1\right)$ (B) $\frac{1}{2}\left(e^{-6}-1\right)$ (C) $e^{-6}+1$ (D) $\frac{1}{2}\left(e^6+1\right)$ (E) e^6-1

5. Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x}+5}{\cos^2 x}\,dx$

(A) $2e+3$ (B) $2e$ (C) $2e-3$ (D) e (E) $e+5$

6. $\int \frac{4}{x}(1+2\ln x)^3\,dx =$

(A) $(1+2\ln x)^4+C$ (B) $\frac{1}{2}(1+2\ln x)^4+C$ (C) $-\frac{1}{2}(1+2\ln x)^4+C$ (D) $\frac{1}{2}\ln x(1+2\ln x)^4+C$

(E) $-(1+2\ln x)^4+C$

11. Evaluate $\int_e^{e^4} \frac{5}{x\sqrt{\ln x}}\,dx$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

12. Evaluate the following indefinite integrals. Don't forget your $+C$.

(a) $\int 2x\left(x^2+1\right)dx$ (b) $\int \frac{3t^2}{t^3-4}\,dt$ (c) $\int x\sqrt{2x^2-1}\,dx$ (d) $\int 3xe^{x^2+2}\,dx$

Differential Equations Review.

3. (Use your calculator on this one, too, but get the exact answer first.) The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time t. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?

(A) 3.750 (B) 4.000 (C) 4.066 (D) 4.132 (E) 4.600

$\frac{dP}{dt} = kP$ (P = # of acres consumed by a plot of kudzu)

→ seperate variables.

Free Response

For problems 4 – 13, find the general solution to the following differential equations, then find the particular solution using the initial condition.

4. $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = -2$

5. $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = 3$

6. $\frac{dy}{dx} = \frac{y}{x}$, $y(2) = 2$

7. $\frac{dy}{dx} = 2xy$, $y(0) = -3$

8. $\frac{dy}{dx} = (y+5)(x+2)$, $y(0) = -1$

9. $\frac{dy}{dx} = \cos^2 y$, $y(0) = 0$